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Stability and Quasi Normal Modes of Charged Born–Infeld Black Holes

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The static charged Born–Infeld black hole is stable. We compare its stability to the linear counterpart Reissner–Nordstrom black hole stability. We use the WKB method to compute how its quasi-normal modes vary with the non-linear parameter, temperature, mass of the scalar field and the spherical index.

KEY WORDS: static; charged; Born–Infeld; black holes; quasi-normal modes.

1. INTRODUCTION

In this paper we focus on a black hole arising in Einstein–Born–Infeld gravity. Born–Infeld electrodynamics was first introduced in 1930's to obtain a classical theory of charged particles with finite self-energy (Born and Infeld, 1934). In Maxwell's theory of electrodynamics, the field of a point-like charge is singular at the origin and its energy is infinite. In contrast, in the Born–Infeld electrodynamics, the electric field of a point like object which is given by $E_r = Q/\sqrt{r^4 + \frac{Q^2}{\beta^2}}$ is regular at the origin and its total energy is finite. Born–Infeld theory has received renewed attention since it turns out to play an important role in string theory. It arises naturally in open superstrings and in D-branes (Leigh, 1989). The low energy effective action for an open superstring in loop calculations lead to Born–Infeld type actions (Fradkin and Tseytlin, 1985). It has also been observed that the Born–Infeld action arises as an effective action governing the dynamics of vector-fields on D-branes (Tseytlin, 1986). For a review of aspects of Born–Infeld theory in string theory see Gibbons (2003) and Tseytlin ().

If we construct black hole solutions to Einstein–Born–Infeld gravity, it can be observed that the non-singular nature of the electric field at the origin changes the structure of the space-time drastically. The singularity at the origin is dominated

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by the mass term M/r rather than the charge term Q^2/r^2 as in the black holes of Einstein–Maxwell gravity (Chandrasekhar, 1992). Here the possibility exists for M to be positive, negative or zero leading to space-like or time-like singularities in contrast to the Reisser–Nordstrom black hole. Overall, the physical properties of black holes change drastically due to the non-linear nature of electrodyanmics and worthy of study. In this paper, we are focusing on the static charged black holes of the Born–Infeld electrodynamics. This is the non-linear generalization of the Reissner–Nordstrom black hole and is characterized by charge Q, mass M and the non-linear parameter β. The Born–Infeld black hole was obtained by Garcia *et al.* (1984) in 1984. Two years later, Demianski (1986) also presented a solution known as EBIon which differs with the one in Garcia *et al.* (1984) by a constant. Trajectories of test particles in the static charged Born–Infeld black hole was discussed by Breton (Breton). This black hole in isolated horizon framework was discussed by the same author in Breton (2003). Gibbons and Herdeiro (2001) derived a Melvin Universe type solution describing a magnetic field permeating the whole Universe in Born–Infeld electrodynamics coupled to gravity. By the use of electric-magnetic duality, they also obtained Melvin electric and dyonic Universes.

Here we will study the stability and quasi normal modes of the Born–Infeld black holes. Recently, there has been a renewed interest in computation of the quasi normal mode (QNM) frequencies of black hole solutions. These are the modes of damped oscillations when black holes are perturbed. The frequencies of QNM's depend on the parameters of the black holes such as mass, charge and angular momentum and are independent of initial perturbations. If the radiation due to QNM modes are detected in the future by gravitational wave detectors, it would be a clear way of identifying the possible charges of black holes. There are extensive studies of QNM's in various black-hole backgrounds in the literature. See the review by Kokkotas *et al.* (1999) for more information.

There are many reasons to study QNM's. One of them is Loop quantum gravity. In particular, it has been observed that for asymptotically flat black holes, the real part of the high overtones of QNM's coincide with the Barberr–Immirizi parameter γ. The value of γ, fixed via QNM's, turned out to be precisely the one required to make the loop quantum gravity entropy predictions coincide with the classical Bekenstien–Hawking entropy. Some of the work done along these lines are given in the references (Andersson and Howls, 2004; Cardoso *et al.*, 2004; Corichi, 2003; Dreyer, 2003; Hod, 1998; Kunstatter, 2003; Maassen van den, 2004; Motl, 2003; Motl and Neitzke, 2003; Natario and Schiappa; Setare, 2004a,b).

Another important aspect of QNM studies have been related to the conjecture relating anti-de Sitter(AdS) and conformal field theory (CFT) (Aharony *et al.*, 2000). It is conjectured that the imaginary part of the QNM's, which gives the time scale to decay the black hole perturbations, corresponds to the time scale of the CFT on the boundary to reach thermal equilibrium. There are many work on AdS black holes in four and higher dimensions on this subject and we will refer a few among them in Horowitz and Hubeny (2000); Cardoso and Lemos (2001); Wang *et al.* (2000); Konoplya (2002a) .

The paper is presented as follows: In Section 2, the Born–Infeld black hole solutions are introduced. In Section 3, the scalar perturbations are given. In Section 4, we will computer the QNM's and discuss the results. Finally, the conclusion is given in Section 5.

2. STATIC CHARGED BLACK HOLE IN EINSTEIN–BORN–INFELD GRAVITY

In this section, an introduction to the static charged black hole in Einstein–Born–Infeld gravity is given. The most general action for a theory with non-linear electrodynamics coupled to gravity is as follows:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + L(F)\right] \tag{1}$$

Here, $L(F)$ is a function of the field strength $F_{\mu\nu}$ only. In the weak field limit, $L(F)$ has to be of the form

$$L(F) = -F^{\mu\nu}F_{\mu\nu} + O(F^4) \tag{2}$$

In this paper, a particular non-linear electrodynamics called the Born–Infeld theory is studied which has attracted lot of attention due to its relation to string effective actions. The function $L(F)$ for Born–Infeld electrodynamics may be expanded to be

$$L(F) = 4\beta^2\left(1 - \sqrt{1 + \frac{F^{\mu\nu}F_{\mu\nu}}{2\beta^2}}\right) \tag{3}$$

Here, β has dimensions $length^{-2}$ and $G\ length^2$. In the following sections it is assumed that $16\pi G = 1$. Note that when $\beta \to \infty$, the Lagrangian $L(F)$ approaches the one for Maxwell's electrodynamics given by $-F^2$.

Out of all theories of non-electrodynamics, Born–Infeld theory has a special place that it is also invariant under electic-magnetic duality. This is discussed in detail by Gibbons and Rasheed (1995).

For static spherically symmetric case, the electric field is given by

$$F_{\rm tr} = E(r) = -\frac{Q}{\sqrt{r^4 + \frac{Q^2}{\beta^2}}} \tag{4}$$

Note that for this case the non-linear Lagrangian reduces to

$$L(F) = 4\beta^2 \left(1 - \sqrt{1 - \frac{E^2}{\beta^2}}\right) \tag{5}$$

imposing an upper bound for $|E| \leq \beta$. This is a crucial characteristic of Born–Infeld electrodynamics which leads to finite self energy of the electron.

By solving the field equations, the static charged black hole with spherical symmetry can be obtained as

$$ds^2 = f(r)\,dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \mathrm{Sin}^2(\theta)\,d\varphi^2) \tag{6}$$

with

$$f(r) = 1 - \frac{2M}{r} + 2\beta\left(\frac{r^2\beta}{3} - \frac{1}{r}\int_r^\infty \sqrt{Q^2 + r^4\beta^2}\right) \tag{7}$$

The integral in $f(r)$ can be simplified leading to

$$f(r) = 1 - \frac{2M}{r} + \frac{2\beta r^2}{3}\left(1 - \sqrt{1 + \frac{Q^2}{r^4\beta^2}}\right) + \frac{4Q^2}{3r^2}\;{}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{\beta^2 r^4}\right) \tag{8}$$

Here ${}_2F_1$ is the hypergeometric function. In the limit $\beta \to \infty$, the elliptic integral can be expanded to give

$$f(r)_{\mathrm{RN}} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \tag{9}$$

resulting in the function $f(r)$ for the Reissner–Nordstrom black hole for Maxwell's electrodynamics. Near the origin, the function $f(r)$ has the behavior

$$f(r) \approx 1 - \frac{(2M - A)}{r} + 6\beta Q + \frac{2\beta^2}{3}r^2 - \frac{\beta^2}{5}r^4 \tag{10}$$

Here

$$A = \sqrt{\frac{\beta}{\pi}}Q^{3/2}\Gamma\left(\frac{1}{4}\right)^2 \tag{11}$$

Depending on the values of M, Q and β, the function $f(r)$ for the Born–Infeld black hole can have two roots, one root or none. When $f(r)$ has two roots, the behavior of $f(r)$ is similar to the Reissner–Nordstrom black hole. When it has only a single root, the black hole behave similar to the Schwarzschild black hole. Hence the Born–Infeld black hole is interesting since it possess the characteristics of the most well known black holes in the literature. In the following figure we

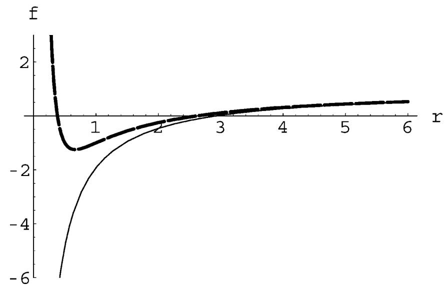

Fig. 1. The figure shows the function $f(r)$ for $M = 1.5$ and $Q = 1$. The lighter graph shows the Born–Infeld black hole and the darker one shows the Reissner–Nordstrom black hole.

have sketched the function $f(r)$ for both Born–Infeld and Reissner–Nordstrom black hole. This is a case where Born–Infeld black hole has a single root (Fig. 1).

A detailed description of the characteristics of the Born–Infeld black hole is given Rasheed and Breton.

The Hawking temperature of the black hole is given by

$$T = \frac{1}{4\pi}\left[\frac{1}{r_+} + 2\beta\left(r_+\beta - \frac{\sqrt{(Q^2 + r_+^2\beta^2)}}{r_+}\right)\right] \tag{12}$$

Here, r_+ is the event horizon of the black hole which is a solution of $f(r) = 0$. The zeroth and the first law of black holes in Born–Infeld electrodynamics are discussed in detail in Rasheed.

Static charged black hole solution to the above action with a cosmological constant was presented in Fernando and Krug (2003); Cai *et al.* (2004) and was extended to higher dimensions by Dey (2004).

3. SCALAR PERTURBATION OF CHARGED BORN–INFELD BLACK HOLES

In this section, we will develop the equations for a scalar field in the background of the static charged black hole introduced in the previous section. The general equation for a massless scalar field in curved space-time can be written as,

$$\nabla^2\Phi = 0 \tag{13}$$

which is also equal to

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi) = 0 \tag{14}$$

Using the ansatz

$$\Phi = e^{-i\omega t} Y(\theta, \phi) \frac{\xi(r)}{r} \tag{15}$$

Equation (21) leads to the radial equation

$$\left(\frac{d^2}{dr_*^2} + \omega^2 \right) \xi(r) = V(r_*) \xi(r) \tag{16}$$

where

$$V(r) = \frac{l(l+1)}{r^2} f + \frac{f f'}{r} \tag{17}$$

and r_* is the well known "tortoise" coordinate given by

$$dr_* = \frac{dr}{f} \tag{18}$$

Since $f(r)$ is not in closed from, r_+ cannot be evaluated explicitly. Note that l is the spherical harmonic index. Hence, when $r \to \infty$, $r_* \to \infty$ and when $r \to r_+$, $r_* \to -\infty$.

The effective potential V for the Born–Infeld black hole is plotted to show how it changes with charge Q and the non-linear parameter β in the following figures (Figs. 2 and 3).

3.1. Remarks on Stability

The potentials are real and positive outside the event horizon as it is evident from the above figures. Hence, following the arguments by Chandrasekhar (1992)

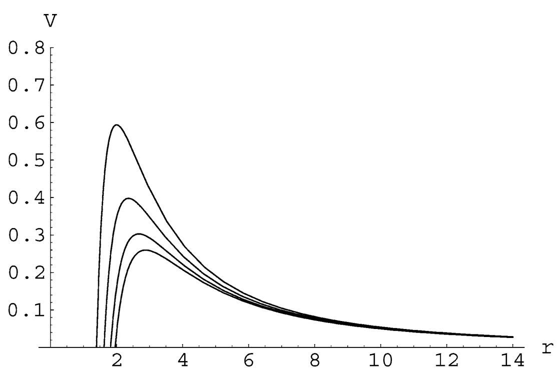

Fig. 2. The behavior of the effective potential $V(r)$ with the charge for the Born–Infeld black hole. Here, $M = 1$, $\beta = 0.2$, and $l = 2$. The height of the potential decreases when the charge decreases.

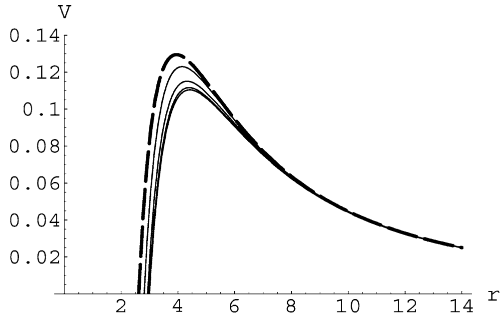

Fig. 3. The behavior of the effective potential $V(r)$ with the non-linear parameter β. Here, $M = 1.5$, $Q = 1$, and $l = 2$. The maximum height of the potential increases as β increases. The dashed one is the potential for the Reissner–Nordstrom black hole with same mass and charge.

the Born–Infeld black holes can be considered stable classically under scalar perturbations. Note that in Fernando (2005), it was shown to be stable for gravitational perturbations.

4. QUASI-NORMAL MODES OF THE BORN–INFELD BLACK HOLE

Quasi-normal modes (QNM) of a classical perturbation of black hole space-times are defined as the solutions to the related wave equations characterized by purely ingoing waves at the horizon. In addition, one has to impose boundary conditions on the solutions at the asymptotic regions as well. In asymptotically flat space-times, the second boundary condition is for the solution to be purely outgoing at spatial infinity. Once these boundary conditions are imposed, the resulting frequencies become complex and discrete.

Usually, the fundamental equation of black hole perturbations given in Eq. (23) cannot be solved analytically. This is the case for almost all black holes in $3 + 1$ dimensions. In $2 + 1$ dimensions, there are two black hole solutions (BTZ black hole (Birmingham, 2001) and the charged dilaton black hole (Fernando, 2004)) which can be solved to give exact values of QNM's. In five dimensions, exact values are obtained for vector perturbations by Nunez and Starinets (2003).

There are several approaches to compute QNM's in the literature. Here, a semi analytical technique developed by Iyer and Will (1987) is followed. The method makes use of the WKB approximation. The basics of this method is reviewed in Fernando (2005). This approach has been applied to compute QNM's of many black holes. Scalar perturbations of charged dilaton black hole (Fernando and Arnold, 2004), charged scalar perturbations of Reissner–Nordstrom black hole

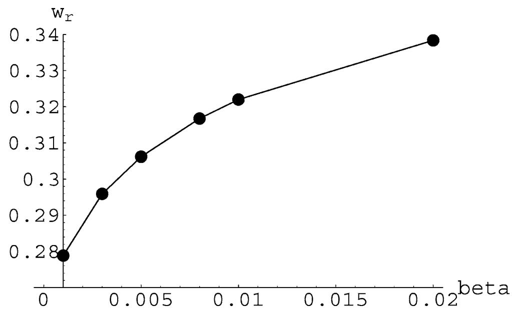

Fig. 4. The behavior of Re ω with the non-linear parameter β for $M = 2$, $Q = 1$, and $l = 2$.

(Konoplya, 2002b) and scalar perturbations of acoustic black holes (Berti *et al.*, 2004) are a few of such studies.

Let ω be represented as $\omega = \omega_R - i\omega_I$. The lowest quasi normal modes $\omega(0)$ of the Born–Infeld black holes are computed.

First, the quasi normal modes are computed to see the behavior with the non-linear parameter β and graphed in the following figures (Figs. 4 and 5).

Next, we have computed the QNM's by varying the charge of the black hole. We also computed the QNM's for the Reissner–Nordstrom black hole with the same mass and the charge. Both results are plotted in the following figures (Figs. 6 and 7).

For larger values of the Q, the decay of the scalar field is faster. From the above plots, one can observe that the decay of the scalar field in Born–Infeld black hole is faster than the Reissner–Nordstrom black hole for this particular values of the parameters. One can also conclude that the Born–Infeld black hole are stable since the fields decay in this background. Note that we have done these computations for a Born–Infeld back hole which behaves like the Schwarzschild black hole near the origin. It is necessary to do a evaluation based on all the

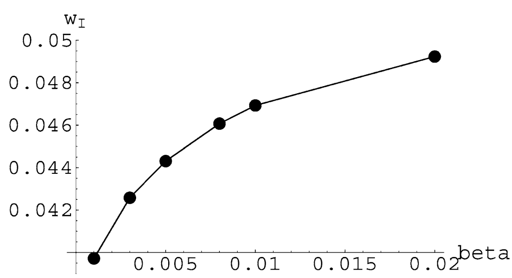

Fig. 5. The behavior of Im ω with the non-linear parameter β for $M = 2$, $Q = 1$, and $l = 2$.

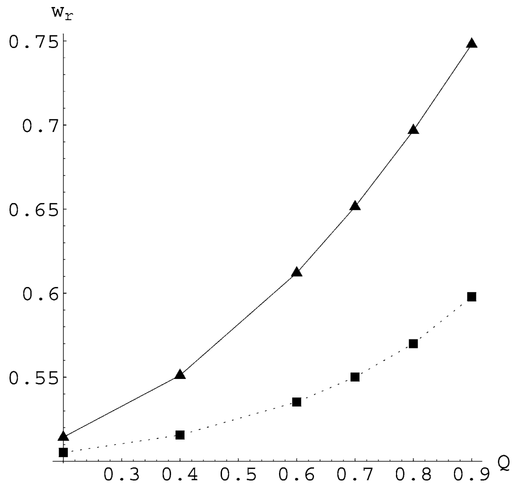

Fig. 6. The behavior of Re ω with the charge Q for $M = 1$, $\beta = 0.04$, and $l = 2$. The dotted lines show the graph for Reissner–Nordstrom and the dark lines show the one for the Born–Infeld black hole.

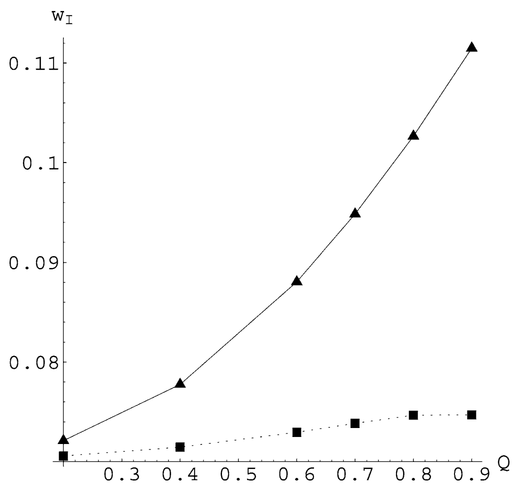

Fig. 7. The behavior of Im ω with the charge Q for $M = 1$, $\beta = 0.04$, and $l = 2$. The dotted lines show the graph for Reissner–Nordstrom and the dark lines show the one for the Born–Infeld black hole.

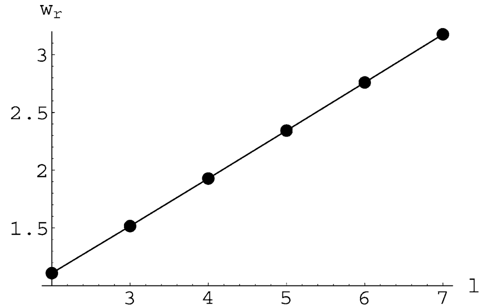

Fig. 8. The behavior of Re ω with the spherical index l for $M = 2$, $Q = 1$, and $\beta = 1$.

parameters to fully understand the behavior of the ω_{I} to see how the stability compare with the Reissner–Nordstrom black hole.

We also studied the behavior of the quasi normal modes with spherical index l as given in the following figure (Figs. 8 and 9).

It is observed that when the spherical index l is increased, the Re ω increases and Im ω approaches a fixed value. This is similar to the behavior observed for charged scalar dilaton black holes in Fernando and Arnold (2004).

Next, we have studied the behavior of the Im ω with temperature of the black hole. The plot of Im (ω) vs temperature is given in the Fig. 9. One can see a linear behavior of Im ω with the temperature. This behavior is similar to the Schwarzschild-anti-de-Sitter black hole studied by Horowitz and Hubeny (2000) (Fig. 10).

4.1. Asymptotic Quasi Normal Modes

One of the reasons to attract attention to QNM's computation in the recent past has to do with the conjecture that the real part of QNM's corresponds to the

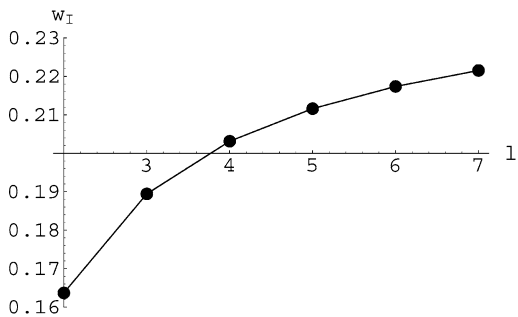

Fig. 9. The behavior of Im ω with the spherical index l for $M = 2$, $Q = 1$, and $\beta = 1$.

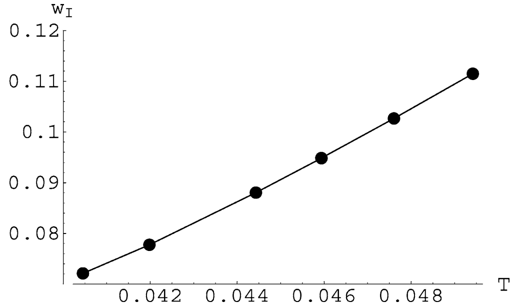

Fig. 10. The behavior of ω_{I} with the Temperature for $l = 2$. The mass $M = 1$ and $\beta = 0.04$.

minimum energy change of a quantum transitions of a black hole Hod (1998); Dreyer (2003). There have been a number of works done to compute the high frequency QNM's and the asymptotic value of the real part of QNM frequencies. Recently, Das and Shankaranarayanan developed a method to compute QNM frequencies of $(D + 2)$ dimensional spherically symmetric single horizon black holes with generic singularities (Das and Shankaranarayanan, 2005). In this paper, we apply this method to compute the asymptotic values of the QNM's of the Born–Infeld black hole. First, we will summarize their derivation as follows: Consider a general $(D + 2)$ dimensional spherically symmetric line element given by

$$ds^2 = f(r)\,dt^2 + \frac{dr^2}{g(r)} + r^2\,d\Omega^2 \tag{19}$$

Suppose there is a singularity of the line element at $r \to 0$, then near the singularity the function $f(r)$ and $g(r)$ have the power law behavior as

$$f(r) \approx r^{\frac{2p}{q}}$$
$$\frac{1}{g(r)} \approx r^{\frac{2(p-q+2)}{q}} \tag{20}$$

This was shown in the works of Szekeres–Iyer (1993; 2002). For scalar perturbations, Das and Shankaranarayanan obtained

$$\omega_{\mathrm{QNM}} = \frac{i}{2c_0}\left(n + \frac{1}{2}\right) \pm \frac{1}{4\pi c_0}\mathrm{Log}\left(1 + 2\,\mathrm{Cos}\left(\frac{\pi}{2}(Dq - 2)\right)\right) \tag{21}$$

Here, c_0 is a constant and is related to the temperature. For the Born–Infeld black hole $c_0 = \frac{1}{4\pi T}$.

Now, we will apply this method to compute the QNM's of the Born–Infeld black hole. Recall that closer to the singularity $r \to 0$, the function $f(r)$ is given in Eq. (17). Hence, a power law behavior for $f(r)$ and $g(r)$ can be obtained as

$$f(r) \approx a_1 r^{-1} \Rightarrow \frac{2p}{q} = -1 \tag{22}$$

Also,

$$\frac{1}{g(r)} = \frac{1}{f(r)} \approx \frac{r}{a_1} \Rightarrow 2\left(\frac{p}{q} - 1 + \frac{2}{q}\right) = 1 \tag{23}$$

Note that $a_1 = 2M - A$. Now by comparing Eqs. (34) and (35) with Eq. (33), p and q values for the Born–Infeld black hole can be computed to be

$$p = -\frac{1}{2}; \qquad q = 1 \tag{24}$$

From Eq. (33), the QNM's are given by

$$\omega_{\rm QNM} = i2\pi T_{\rm H}\left(n + \frac{1}{2}\right) \pm T_{\rm H}\,{\rm Log}(3) \tag{25}$$

The real part of ω behaves as $T_{\rm H}{\rm Log}(3)$ according to this derivation. In fact in Das and Shankaranarayanan (2005) it was derived that $D+2$ dimensional Schwarzschild, and stringy black holes in four and five dimensions also have the same behavior for real ω.

4.2. Massive Scalar Perturbations

It has been observed that the massive modes decay slower than the massless field for the Schwarzschild black hole in Burko and Khanna (2004); Konoplya and Zhidenko (2005). An interesting question was posed whether the decreasing decay rate can approach zero leading to a long living mode. Here we are computing the QNM's for a massive scalar field in the Born–Infeld black hole to see if this behavior persist in this case.

The general equation for a massive scalar field in curved space-time is written as

$$\bigtriangledown^2 \Phi - m^2 \Phi = 0 \tag{26}$$

Using the ansatz

$$\Phi = e^{-i\omega t} Y(\theta, \phi) \frac{\xi(r)}{r} \tag{27}$$

Equation (38) leads to the radial equation given in Eq. (23) with the modified potential

$$V(r) = \frac{l(l+1)}{r^2} f + \frac{f f'}{r} + m^2 f(r) \tag{28}$$

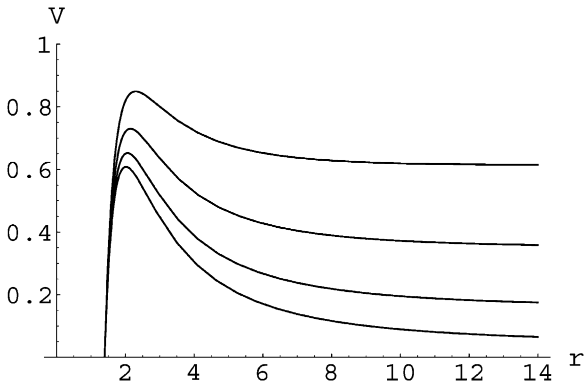

Fig. 11. The behavior of V with the mass m of the scalar field. Here $M = 1$, $Q = 0.8$, $\beta = 0.2$, and $l = 2$. When the mass decreases, the height of the potential also decreases.

The effective potential V is plotted by varying the mass m of the scalar field in the following figure (Fig. 11).

We computed the QNM's for the massive scalar field decay using the WKB approximation discussed in Section 4. They are given in the following figures (Figs. 12 and 13).

By observing the behavior of Im ω when the mass m increases it is clear that the decay is slower for massive scalar field similar to the Schwarzschild case. On the other hand, the frequency of oscillations of the modes seems to increase with the mass.

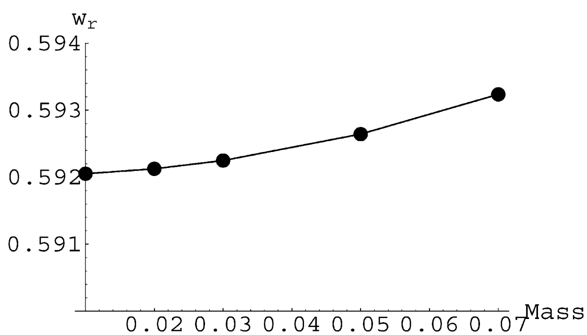

Fig. 12. The behavior of Re ω with the mass of the scalar field m for $M = 1$, $Q = 1$, $\beta = 0.04$, and $l = 2$.

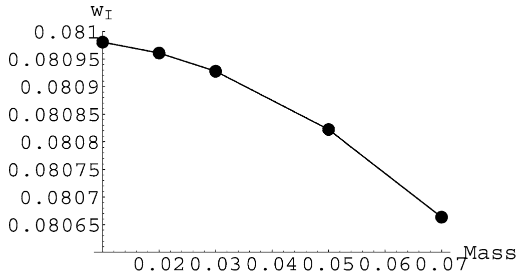

Fig. 13. The behavior of Im ω with the mass of the scalar field m for $M = 1$, $Q = 1$, $\beta = 0.04$, and $l = 2$.

5. CONCLUSION

We have studied quasi normal modes for the charged black holes in Einstein–Born–Infeld gravity with scalar perturbations. The lowest quasi normal modes are computed using a WKB method.

The main result of this paper is that Born–Infeld black holes are stable under scalar perturbations. It is also noted that the scalar field decay faster in the Born–Infeld black hole background in comparison with the Reissner–Nordstrom black hole. We have done this study only for a particular class of parameters and a detailed analysis is necessary to predict the behavior for all parameters.

We obtained an approximation to the asymptotic QNM's using a method developed in Das and Shankaranarayanan (2005). There, the linear behavior of ω with the temperature was observed. The computed ω_{I} using the WKB method support this. Similar behavior was observed first by Horowitz and Hubeny for Schwarzschild-anti-de-Sitter black holes (Horowitz and Hubeny, 2000). There, they showed a linear relation between QNM's and temperature for large black holes in several dimensions. For black holes, in anti-de-Sitter space, relations between QNM's and conformal field theory of the boundary are discussed in many papers.

It was also noted that when the spherical index l is increased, the Re ω increases leading to greater oscillations. On the other hand, Im ω approaches a fixed value for larger l. This is similar to what was observed in the static charged dilaton black hole in Fernando and Arnold (2004).

Even though the paper's focus was on massless scalar field, we also analyzed the behavior of a massive scalar field. It decays slower than the massless field, similar to the observation of other black holes (Konoplya and Zhidenko, 2005).

A natural extension of this work is to study the stability of the Born–Infeld black hole with a cosmological constant (Fernando and Krug, 2003). In particular, the Born–Infeld-Ads case is worthy of study due to the AdS/CFT conjecture. The electrically charged black hole in anti-de Sitter space has been shown to be unstable for large black holes by using linear perturbation techniques (Gubser and Mitra). It would be interesting to study how the non-linear nature effects the instability of such solutions.

It would also be interesting to investigate the supersymmetric nature of the Born–Infeld black hole discussed in this paper. It is well known that the extreme Reissner–Nordstrom black hole can be embedded in $N = 2$ supergravity theory (Gibbons and Hull, 1982; Gibbons, 1982). Onozawa *et al.* (1997) showed that the QNM's of the extreme RN black hole for spin 1, 3/2, and 2 are the same. A suitable supergravity theory to embed the Born–Infeld black hole has not been constructed. However, one can do a similar study such as in Onozawa *et al.* (1997) in terms of the QNM's for extreme Born–Infeld black holes. From the Eq. (17) when $2M - A$ is smaller than zero, the black holes behave similar to the Reissner–Nordstrom black hole. Hence, it is possible to construct extreme case for Born–Infeld black hole by choosing appropriate parameters.

The WKB approach considered here was extended to the sixth order by Konoplya (2003) which gives greater accuracy in computing the QNM frequencies. It would be interesting to use it to compute frequencies of high overtones.

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